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tion to this rule. In fact from the great variety of subjects treated, the task of their authors is an unusually difficult one. It is therefore important that the reader should be on his guard and utilize all available material instead of relying completely on some one author.

Stanford University, November, 1902.

A DEVELOPMENT OF THE CONIC SECTIONS BY KINEMATIC METHODS.

By JOHN JAMES QUINN, Ph. B., Head of the Department of Mathematics and Manual Training,
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THE CIRCLE.

PROPOSITION I. *If two lines A and B, pivoted at P and P', respectively, be placed at any angle μ to each other, and both revolve in the same direction with the same angular velocity, the locus of their intersection is a circle.*

Let ϕ and ψ denote the angles which A and B in their initial position make with the line PP'. Then

$$\omega + \phi + \mu = 180^\circ = \pi.$$

In any new position of A and B,

$$\omega - \theta + \phi + \theta + \mu' = \pi.$$

Hence $\mu = \mu' = \text{constant}$, so that the locus is a circle.

Remark. If p denotes the distance PP' between the pivots and C denotes the area of the generated circle, then

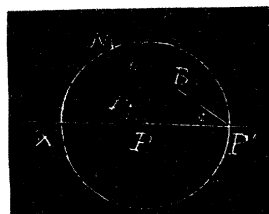
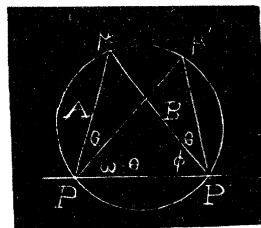
$$C = \pi p^2, \text{ if } \mu = 30^\circ; \quad C = \frac{1}{2}\pi p^2, \text{ if } \mu = 45^\circ;$$

$$C = \frac{1}{3}\pi p^2, \text{ if } \mu = 60^\circ; \quad C = \frac{1}{4}\pi p^2, \text{ if } \mu = 90^\circ.$$

PROPOSITION II. *If two lines A and B, pivoted in an axis X, and initially coincident with it, revolve in the same direction, the one having twice the angular velocity of the other, then the locus of their intersection is a circle.*

Let ϕ be the angle through which B moves in a unit of time; 2ϕ the angle through which A moves in the same time, and θ the vertical angle.

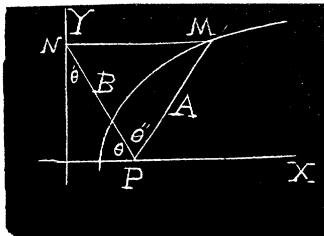
By the conditions, $2\phi = \phi + \theta$. Therefore $\theta = \phi$. Whence the triangle PMP' is isosceles. But the side PP' is constant. Therefore the side PM is constant. Therefore the locus of the intersection of A and B is a circle.



THE PARABOLA.

PROPOSITION III. *If two lines A and B , be pivoted in an axis X , at the same point P , and both revolve in the same direction, but the angular velocity of the line A is to the angular velocity of the line B as 2:1, the locus of the intersection M of A with a line NM parallel to the axis through the point of intersection of B with a fixed perpendicular Y to the axis is a parabola.*

A revolves through 2θ ; B revolves through θ . Since MN is parallel to X , $\theta = \theta' = \theta''$. Therefore $MP = MN$. Therefore the locus of M is a parabola.



THE ELLIPSE.

PROPOSITION IV. *If at two fixed points three lines A , B , and C , be pivoted, A at one point P' , revolving at any velocity, and generating the directing circle with center P' ; B and C at the other point P , revolving in the same direction, but the line C at such a rate that its intersection with A lies in the directing circle; and B at such a rate that the angle BC is constantly equal to the angle AC , then the locus of the intersection M of B and A is an ellipse.*

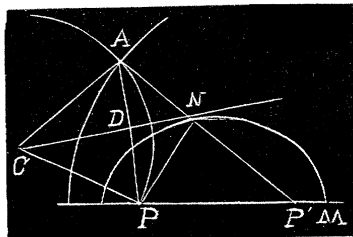
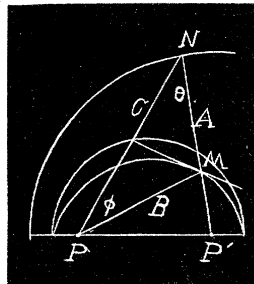
Let the angle $AC = \theta$, and the angle $BC = \phi$. Since the angle $\phi = \text{angle } \theta$, the line $MN = \text{line } PM$. Therefore $PM + P'M = P'N + MN$. But $P'M + MN = A = \text{a constant}$, being the radius of the directing circle. Therefore the locus of M is an ellipse.

COROLLARY. The perpendicular bisector of the line C is tangent to the ellipse at the point M .

PROPOSITION V. *If a circle of center C be described in the plane of an ellipse MN with its circumference passing through a focus P and cutting the directing circle in A and A' , then the lines drawn from C to the mid-points D and D' of PA and PA' are tangent to the ellipse.*

It is to be shown that CD and CD' (not drawn in the figure) are tangents to the ellipse with the foci P and P' . Let N be the intersection of $P'A$ with the ellipse. Then $P'N + NA = P'N + PN$, so that $NA = PN$. But CD is perpendicular to PA at its mid-point, since $CA = CP$. Hence CD passes through the vertex N . By the preceding corollary, CD is tangent to the ellipse at the point N . Similarly for the tangent CD' .

As a corollary, we derive the following construction for the tangents to an ellipse from any external point C in its plane: With C as a center, and CP as a radius, describe an arc cutting the directing circle in A and A' . Draw $P'A$ and $P'A'$ and let them intersect the ellipse at N and N' . Then CN and CN' are the required tangents.



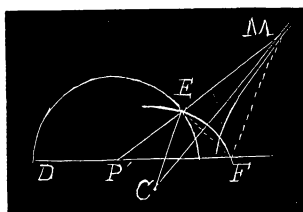
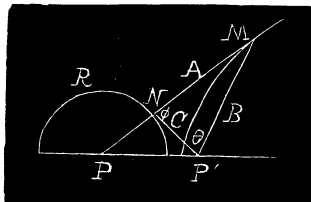
A simple modification leads to a construction for a tangent to a parabola from any external point C . We have only to replace the directing circle by the directrix of the parabola.

THE HYPERBOLA.

PROPOSITION VI. *If at two fixed points P and P' , three lines A , B , and C , be pivoted, A at one point revolving in one direction at any velocity; B and C at the other pivot revolving in an opposite direction, C at such a rate that it constantly intersects A in the circumference of a directing circle described with P as a center, B at such a rate that the angle BC is constantly equal to the angle CA , then the locus of the intersection M of A and B is a hyperbola.*

Let the angle AC be denoted by ϕ and BC by θ . Since $\phi = \theta$, the segment $NM = \text{segment } P'M$ in any position. Therefore $PM - P'M = PM - NM = PN = \text{constant}$. Therefore the locus of M is a hyperbola.

PROPOSITION VII. *If a circle with center C be described in the plane of a hyperbola passing through one focus P and intersecting the directing circle at E , and the*



other focal radius $P'M$ be drawn through this point E to meet the curve, at M , the line CM is tangent to the hyperbola.

Draw MP and EP . The triangle EMP is isosceles and CM is perpendicular to the base PE at its mid-point A . Therefore it passes through the vertex M and is tangent to the hyperbola.

A METHOD FOR CONSTRUCTING AN HYPERBOLA, GIVEN THE ASYMPTOTES AND A FOCUS.

By ARCHIBALD HENDERSON, Ph. D., Associate Professor of Mathematics, University of North Carolina, Chapel Hill, N. C.

Consider any circle, whose center is the point $(0, y_0)$ and whose radius is the distance from this point to the focus $[1/(a^2 + b^2), 0]$ of an hyperbola. The equation of this circle is

$$x^2 + (y - y_0)^2 = y_0^2 + a^2 + b^2,$$

$$\text{or } x^2 + y^2 - 2y_0y - (a^2 + b^2) = 0 \dots (1).$$

Now we may represent any point on an asymptote to the hyperbola